Boolean Properties of Sets — Requirements

Library Committee
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Summary. This article contains proofs of the theorems which are obvious if the directive ‘requirements BOOLE;’ will be added to environment declaration of the Mizar article.

MML Identifier: BOOLE.

WWW: [http://mizar.org/JFM/Vol-3/boole.html]

One can prove the following propositions:

1. For every set $X$ holds $X \cup \emptyset = X$.
2. For every set $X$ holds $X \cap \emptyset = \emptyset$.
3. For every set $X$ holds $X \setminus \emptyset = X$.
4. For every set $X$ holds $\emptyset \setminus X = \emptyset$.
5. For every set $X$ holds $X \setminus \emptyset = X$.
6. For every set $X$ such that $X$ is empty holds $X = \emptyset$.
7. For all sets $x, X$ such that $x \in X$ holds $X$ is non empty.
8. For all sets $X, Y$ such that $X$ is empty and $X \neq Y$ holds $Y$ is non empty.

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